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A STUDY OF ENERGY LEVELS IN EVEN-EVEN YTTERBIUM ISOTOPES BY MEANS OF (*d,p*), (*d,t*), AND (*d,d'*) REACTIONS

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CONTENTS

Synopsis

Targets prepared from separated isotopes of all the stable ytterbium nuclei have been bombarded with 12 MeV deuterons from a tandem Van de Graaff accelerator. The reaction products were analyzed with a broad range, single-gap, magnetic spectograph with an overall resolution of 0.1 %. In order to study the levels in even nuclei, the triton and proton spectra from ^{171}Yb and ^{173}Yb targets and the inelastic deuteron spectra from ^{168}Yb, ^{170}Yb, ^{172}Yb, ^{174}Yb, and ^{176}Yb targets were recorded. The systematics of the (d,d') population of vibrational bands are discussed and utilized for the identification of γ- and octupole vibrational levels in nuclei in which these had not previously been observed. The (d,p) and (d,t) cross sections are compared with the predictions of stripping theory of deformed nuclei calculated with empirical values of the single-particle cross sections obtained from experiments on even-even targets. The populations of ground states are in agreement with these predictions as modified by pairing theory. The (d,p) and (d,t) populations of several rotational bands show patterns which resemble those expected for two-quasiparticle states, although in many cases the absolute cross sections were smaller than predicted for a pure configuration. The stripping and pick-up data for the vibrational bands offer strong experimental evidence that these states can be considered as superpositions of two-quasiparticle states. The predicted components are found to be present, but the theoretical amplitude often shows only qualitative agreement with the observed cross sections.

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1. Introduction

The energy spectra of deformed even-even nuclei have mostly been studied in radioactive decay or by Coulomb excitation. These studies have provided evidence for the occurrence of low-lying quadrupole vibrational states and some rather scattered information about octupole vibrational states. In addition, it has been possible to identify certain levels as two-particle states, but the basis for the assignments has often been scanty and very little is known about the purity of such configurations.

The microscopic descriptions[1] of nuclear vibrational motions have made the distinction between "collective" and "single-particle" excitations less sharp. In the simplest approach, a state in an even nucleus is considered to be a superposition of two-quasiparticle states. The "single-particle" excitations have only one such component, whereas the "collective" states are characterized by the presence of many components, each with a small amplitude. However, it is to be expected that states with structure intermediate to those mentioned can occur.

The microscopic treatment of nuclear states is an excellent guide for the interpretation of the results of single-particle transfer reactions and, conversely, the experimental cross sections for such reactions are measures of the amplitudes for certain configurations. Thus, the measurements can be compared directly to theoretical predictions.

It should be stressed that, for even-even final nuclei, only amplitudes involving the odd target ground-state particle can be measured by single-particle transfer reactions. Furthermore, only fairly large amplitudes can be observed by transfer processes. Even if the experimental spectrum is clean and well resolved so that small cross sections can be detected, there will always be considerable uncertainties about other mechanisms which could result in such intensities. This is in contrast to the situation for several types of decay processes in which rather small admixtures often cause appreciable effects.

Collective vibrational states based on the ground state are most easily localised by Coulomb excitation or by various inelastic scattering processes. In many cases, the cross section for such reactions also measures to a good approximation the reduced electric multipole transition probability between the ground state and the excited state.

In the present work, the (d,p), (d,t), and (d,d') reactions are used to investigate the energy levels in the even isotopes of ytterbium. A previous report²⁾ describes the results of similar measurements in the odd ytterbium isotopes. The present investigations are largely based on the earlier findings, and it might therefore be appropriate to present a summary of some of the results for the odd final nuclei.

At lower excitation energies, the cross sections for the neutron transfer processes were consistent with the nuclear states representing almost pure single-particle motion in a deformed well. The cross sections were well accounted for on the basis of the stripping theory developed by SATCHLER³⁾, the NILSSON⁴⁾ wave functions, and single-particle cross sections obtained from a DWBA (Distorted Wave Born Approximation) calculation⁵⁾. The absolute cross sections for the low-lying states clearly showed the effects of a partial filling of the states near the Fermi surface in the target nuclei. These results are in quantitative agreement with the predictions of the pairing theory.

At higher excitation energies, there were several indications of a considerable mixing of states, the most noticeable being the absence of cross sections as large as those predicted for pure single-particle states. This breakdown of the single-particle description seemed to take place at an energy approximately corresponding to the quadrupole phonon energy. A related phenomenon was a considerable spreading of quadrupole vibrational strength. This was indicated by the absence, in any single band populated by the (d,d') process, of a cross section comparable to that found in the even nuclei.

Although many of the low-lying states of deformed odd nuclei are well described by the Nilsson wave functions, it appears that the odd nuclei of Yb offer particularly favourable examples of pure single-particle motions. This is partly related to the fact that, in these nuclei, the collective vibrational modes are weak and are found at high excitation energies and partly to the absence of close-lying single-particle levels with K differing by 0 or 1 unit.

The present study of the levels in the even isotopes of Yb has largely profited from the knowledge of the single-particle states in the odd isotopes. Also the general confidence in methods and procedures which have been subject to an experimental test has been important for the experimental study of levels in even-even final nuclei, which is more difficult for several reasons. The transferred particle can couple with the odd-target particle to form two different configurations, that is two rotational bands, as compared to one band for the odd final nucleus. The level density above the energy gap in an even-even nucleus is thus greater than that near the ground state in an odd nucleus, which means that better resolution is desirable. There are additional experimental difficulties because each particle group can contain contributions with different l-values. The intensity patterns for rotational bands populated in a given single-particle transfer process are characteristic of the intrinsic states on which the bands are based. However, these patterns are often not as unique as in the case of odd final nuclei because of the averaging effects introduced by the summation of several components to form the cross section for each final state. Finally, there is the practical consideration that only one or two stable odd targets exist for a given atomic number, a fact which limits the number of single-particle transfer processes that can be used to populate states in even-even nuclei.

On the other hand, for the study of the even nuclei, empirical values of the single-particle cross sections obtained from the studies of odd nuclei could be used to improve the predictions for the single-particle cross sections. This method has been widely applied in the present paper. The interpretation of the inelastic deuteron scattering data is also much simpler for even than for odd nuclei and has been helpful in locating the collective quadrupole and octupole vibrations.

2. Theoretical Considerations

SATCHLER[3)] has given an expression for the cross section of the reaction which transfers a nucleon with angular momentum j to or from a target nucleus of spin I_i to a final state I_f, where I_f is a member of a rotational band in a deformed nucleus. It has been found that the populations of many levels in odd Yb nuclei by (d,p) and (d,t) reactions are well accounted for by this simple description of the process.

For the even target nuclei, the effects of pairing were taken into account simply by multiplying the cross section $(d\sigma/d\Omega)_s$ predicted by Satchler (cf. Eq. 1 of ref. 2) by a factor of U^2 for a (d,p) process and a factor of V^2 for a (d,t) process. These factors are in agreement with the intuitive picture that the cross section for putting a particle into a given state is proportional to the probability for the state being empty and, correspondingly, the cross section for removing a particle is proportional to the probability for the state being full.

The same factors apply for reactions on odd targets except when the transferred neutron is in the same Nilsson orbit as the ground-state particle in the odd target. As an example of this exception, one can consider the (d,t) reaction leading into the ground state of an even-even nucleus. The probability of finding the unpaired neutron in the ground-state orbital, ν, of the odd target nucleus is unity, and the cross section for picking up this neutron is $(d\sigma/d\Omega)_s$. Following this process there is a condition of no neutrons in the state ν which, however, is not the ground state of the even nucleus, since the latter has only a probability $U^2(\nu)$ of state ν being empty. Hence, the cross section $(d\sigma/d\Omega)_s$ should be multiplied by the factor $U^2(\nu)$ which is the overlap of the two cases. Similar reasoning will yield a factor $V^2(\nu)$ in the (d,p) cross section leading to the ground state of an even-even nucleus. One can also invoke the principle of detailed balance to show that these are the appropriate factors to be used for transitions ot the ground states of even-even nuclei. For example, the cross section for the (d,p) reaction between the ground states of the even-even nucleus A and the odd nucleus $A+1$ has a factor $U^2(\nu)$. The (d,t) reaction between the same two ground states can be regarded as the inverse reaction and thus should also be proportional to $U^2(\nu)$. A more formal derivation of these factors is outlined below.

Let us assume that an even-even nucleus has the Bardeen-Cooper-Schrieffer ground state

$$|v = 0\rangle = \prod_{\nu}(U(\nu) + V(\nu)a^{+}(\nu)a^{+}(\bar{\nu}))\,|0\rangle, \tag{1}$$

where $|0\rangle$ and $|v = 0\rangle$ represent the vacuum states for particles and quasiparticles, respectively. Here, ν and $\bar{\nu}$ represent conjugate nucleons while $a(\nu)$ and $a^{+}(\nu)$ are annihilation and creation operators for particles in the state ν. The quasiparticle annihilation and creation operators $\alpha(\nu)$ and $\alpha^{+}(\nu)$ are related to the operators $a(\nu)$, $a^{+}(\nu)$ by the unitary transformation

$$\left.\begin{aligned} \alpha^{+}(\nu) &= U(\nu)a^{+}(\nu) - V(\nu)a(\bar{\nu}) \\ \alpha^{+}(\bar{\nu}) &= U(\nu)a^{+}(\bar{\nu}) + V(\nu)a(\nu). \end{aligned}\right\} \tag{2}$$

If one starts with an odd target nucleus with one quasiparticle in the state ν

$$\alpha^{+}(\nu)\,|v = 0\rangle$$

and removes a nucleon from the same state by means of a pick-up reaction, the result is

$$a(\nu)\ \alpha^{+}(\nu)\,|v = 0\rangle. \tag{4}$$

Thus, the cross section for performing this process and arriving at the ground state of the final nucleus is

$$|\langle v = 0 \mid a(\nu)\alpha^{+}(\nu) \mid v = 0 \rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_s = U^2(\nu)\left(\frac{d\sigma}{d\Omega}\right)_s . \tag{5}$$

The cross sections for stripping and pick-up reactions of this type can be summarized[6] by the relationships

$$\left[\frac{d\sigma}{d\Omega}(j + I_1(v = 0) \longleftrightarrow I_2(v = 1))\right]^{1/2} = U(\nu)\left(\frac{d\sigma}{d\Omega}\right)_s^{1/2} \tag{6a}$$

$$\left[\frac{d\sigma}{d\Omega}(j + I_1(v = 1) \longleftrightarrow I_2(v = 0))\right]^{1/2} = -V(\nu)\left(\frac{d\sigma}{d\Omega}\right)_s^{1/2} \tag{6b}$$

where the single nucleon with angular momentum j is transferred between two nuclear states with spins I_1 and I_2. The mass of the nucleus with I_2 is one unit greater than that with I_1. The quantity v represents the number of unpaired nucleons of the type being transferred.

3. Experimental Details and Results

The beam of 12 MeV deuterons for the present experiments was obtained from the Niels Bohr Institute's tandem Van de Graaff accelerator. The technique of target preparation and operation of the broad-range magnetic spectrograph has been outlined previously[2]. No attempt has been made to measure angular distributions of the reaction products, but exposures were made at two or three different angles for each reaction studied. Triton and proton spectra from targets of ^{171}Yb and ^{173}Yb are shown in Figs. 1–4, and Figs. 5–9 show inelastic deuteron spectra. The energies and cross sections for population of the levels observed are given in Tables 1–5. For a discussion of the uncertainties involved in these measurements, see ref. 2. The Q-values for stripping and pick-up processes leading to the ground states of the daughter nuclei have already been reported[2]. Proposed level schemes for $^{168, 170, 172, 174, 176}$Yb are shown in Figs. 10–14. The assignments given to the levels in these diagrams are discussed in the following section.

4. Interpretation of the Data

4.1. Ground State Rotational Bands

The energies of levels in the ground state rotational bands of these nuclei have been well established up to spin values of eight or ten[8, 14]. In the present (d, d') experiments, the states up to and including the spin six member are

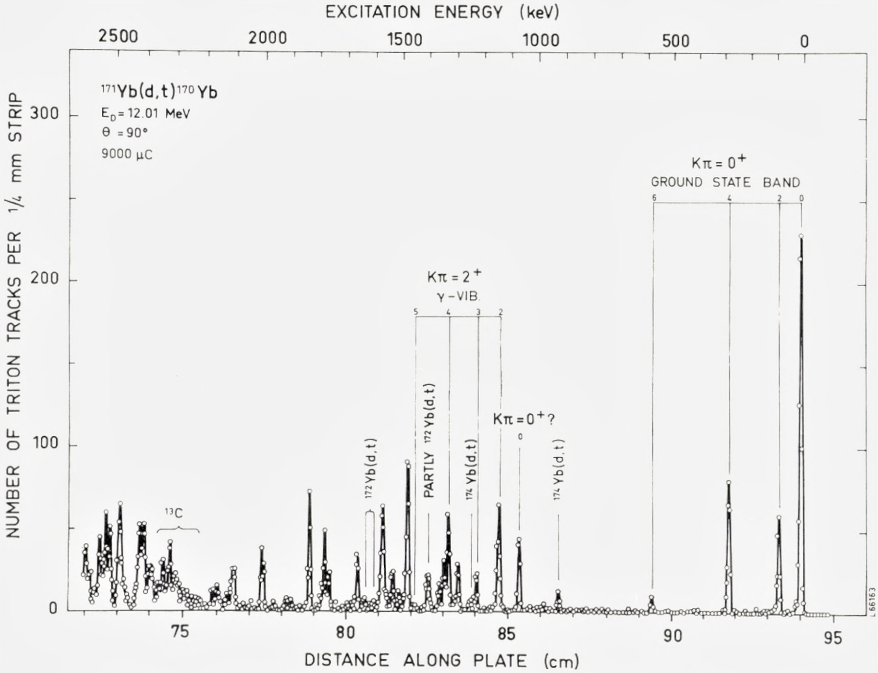

Figure 1. Triton spectrum for the reaction ^{171}Yb (d,t) ^{170}Yb at $\theta = 90°$.

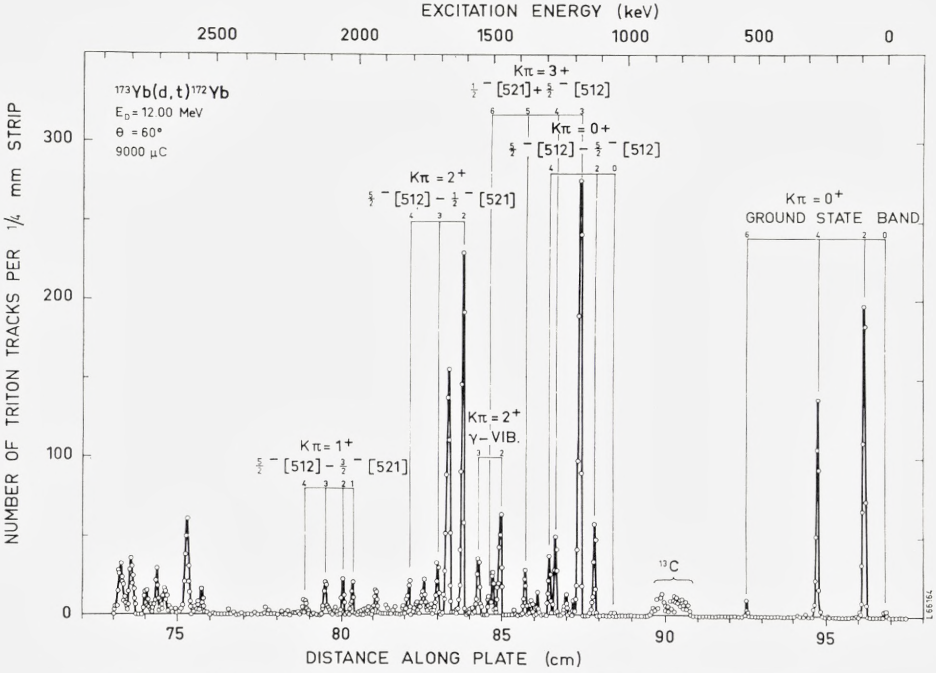

Figure 2. Triton spectrum for the reaction ^{173}Yb (d,t) ^{172}Yb at $\theta = 60°$.

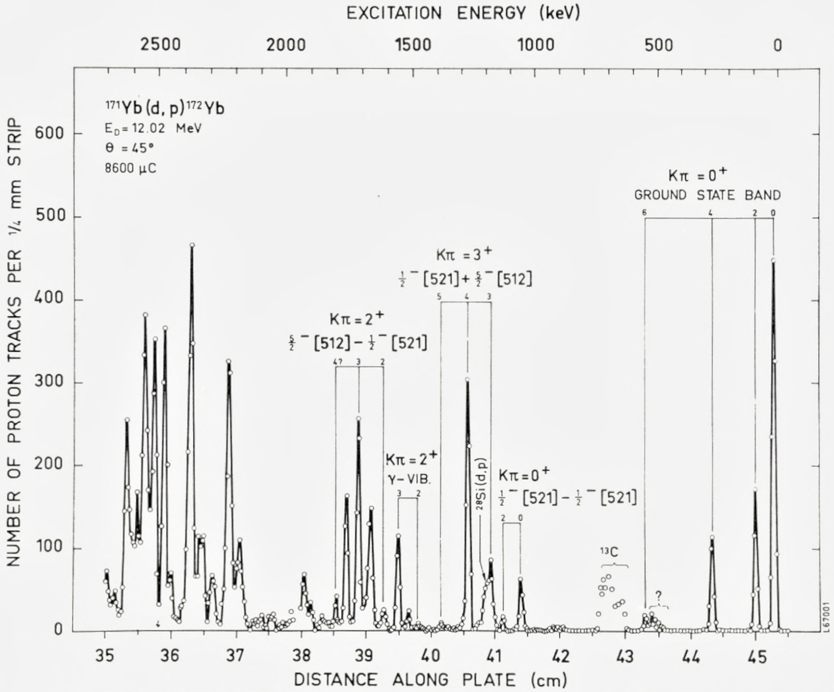

Figure 3. Proton spectrum for the reaction ^{171}Yb (d,p) ^{172}Yb at $\theta = 45°$.

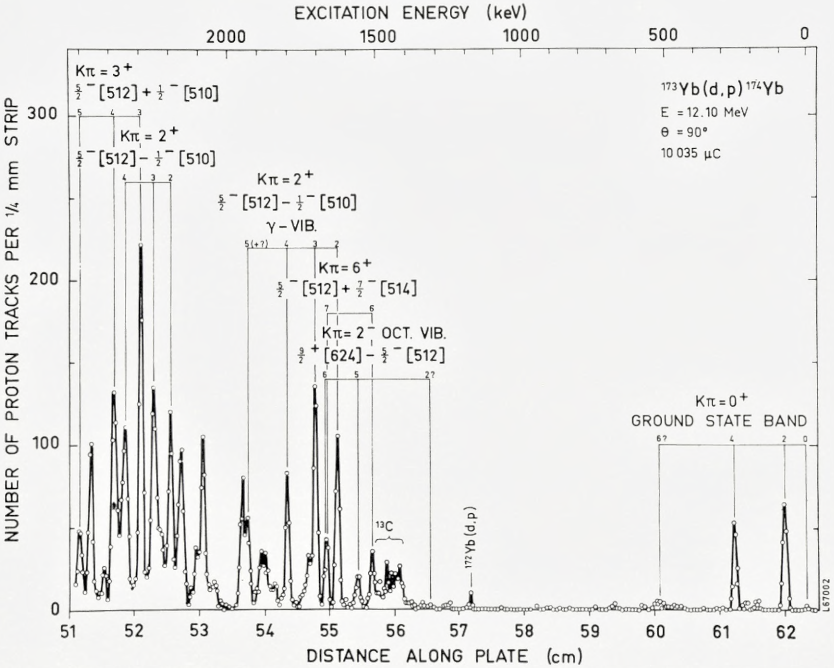

Figure 4. Proton spectrum for the reaction ^{173}Yb (d,p) ^{174}Yb at $\theta = 90°$.

TABLE 1. Levels in ^{168}Yb.

Energy		$d\sigma/d\Omega$ ($\mu b/sr$)		R*	$I, K\pi$	Comments
Previous value[7, 8]	(d,d')	$\theta = 85°$	$\theta = 125°$			
0	0	82600	13000		0,0 +	Ground state rotational band
87.9	87	6300	3090	2.0	2.0 +	
286.9	287	61	69	0.9	4,0 +	
585.7	582	4.3	7.9	0.6	6,0 +	
986	981	127	54	2.3	2,2 +	γ-vibration
	1150	3.3	3.1	1.1	0,0 +	β-vibration
1174	1167	18	27	0.7	4,2 +	γ-vibration
1233	1230	32	8.3	3.9	2,0 +	β-vibration
1391	1387	6.8	7.7	0.9	4,0 +	β-vibration
	1433	2.6	3.0	0.9		
	1475	84	65	1.3	3, ? −	Octupole vibration
	1547	11	12	0.9		
	1595	42	35	1.2	3, ? −	Octupole vibration
	1725	5.1	5.6	0.9		
	1860	12	8.0	1.5		
	1969		5.9			
	1997		3.3			
	2058		13			
	2122		16			

* In this work, R denotes the ratio of (d,d') cross section at $\theta = 90°$ to that at 125°. However, in the case of ^{168}Yb only, the values shown are actually ratios of data at $\theta = 85°$ and $\theta = 125°$. For all levels except the ground state, this will have a very small effect on R due to the slow variation of the (d,d') cross section with θ.

strongly populated. The systematics of the relative populations of the ground state rotational levels in the even rare earth nuclei have been described earlier[15]. It was suggested that the population of the 4+ states could take place in part by a direct E4-transition from the ground state. The Yb-nuclei show particularly low cross sections for the 4+ states, which could indicate that the population is mainly by multiple excitation. In this connection it might be significant that the values of the ratio, R, of the 90° yield to the 125° yield for the Yb-nuclei is less than unity in all cases. In the beginning of the rare earth region where the 4+ cross sections are large, this ratio is[15, 16] approximately 1.20. The 6+ states are also weakly populated in the Yb-nuclei. Strangely enough, in most cases the 90° cross section is larger than the 125° cross section.

The cross sections for the population of levels in the ground state rota-

TABLE 2. Levels in ^{170}Yb.

Energy			$d\sigma/d\Omega$ (d,t) $\mu b/sr$				$d\sigma/d\Omega$ (d,d') $\mu b/sr$		R	$I, K\pi$	Comments
Prev. value ref[8])	(d,t)	(d,d')	$\theta = 45°$	$\theta = 60°$	$\theta = 90°$	$\theta = 125°$	$\theta = 90°$	$\theta = 125°$			
0	0	0	305	222	130	48	63000	13000	4.8	0,0 +	Gr. state rot. band
84.3	84	84	30	45	30	15	5600	2700	2.0	2,0 +	
277.7	275	278	36	60	47	35	37	64	0.6	4,0 +	
	573	577			~ 5	5.6	7.1	5.0	1.4	6,0 +	
	1065		16	25	22	7.1				0,0 + ?	F − F*
	1138	1145	31	47	45	20	77	35	2.2	2,2 +	γ-vib.
	1221		10		14	6.0				3,2 +	γ-vib.
	1289	1300	8.7	19	16	8.7	19	12	1.6		
	1324	1330	8.8	31	32	21	16	22	0.7	4,2 +	γ-v b.
	1341		4.9		16	9.6					
	1358		3.3	12	10	6.1					
	1398	1400	11	17	19	~ 7.5	54	60	0.9	3, ? −	Oct. vib.
	~ 1445				~ 3	2.3				5,2 +	γ-vib.
	1473		43	66	56	29					
	1508		2	13	~ 6	4.4					
	1529		6.5		15	7.8					
	1552			26	~ 5	2.5					
	1564		14		45	35					
		1586					2.3				
	1655		4.7	14	16	11					
	1690			~ 2	~ 2						
	1757			26	12	12					
	1774		~ 6		27	14					
		1783					46	53	0.8	3, ? −	Oct. vib.
	~ 1789		~ 2			~ 3					
	1829		13	25	32	19					
	1911		~ 0.6		4.5						
	1963	1968	1.8		~ 3	~ 1.6	0.8	1.6			
	2000		4.2		24	9.3					
	2106		5.4		21	14					
		2115						7.5			

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* The notation is patterned after that of ref. 20. The letter F indicates the single-particle state nearest to the Fermi level. The level order is as in Fig. 21 of ref. 2, and the key to Tables 2–4 is as follows: F − 4: 3/2 − [521], F − 3: 5/2 + [642], F − 2: 5/2 − [523], F − 1: 7/2 + [633], F: 1/2 − [521], F + 1: 5/2 − [512], F + 2: 7/2 − [514], F + 3: 9/2 + [624], F + 4: 1/2 − [510], F + 5: 3/2 − [512].

TABLE 3. Levels in ^{172}Yb.

Energy				$d\sigma/d\Omega$ (d,t) μb/sr		$d\sigma/d\Omega$ (d,p) μb/sr		$d\sigma/d\Omega$ (d,d′) μb/sr		R	$I, K\pi$	Comments
Prev. value ref.[9,10])	(d,t)	(d,p)	(d,d′)	$\theta = 60°$	$\theta = 90°$	$\theta = 45°$	$\theta = 90°$	$\theta = 90°$	$\theta = 125°$			
0	0	0	0	3	~ 4	106	37	63000	13000	4.8	0,0 +	Gr. state rot. band
79	78	80	79	170	136	34	17	6400	3200	2.0	2,0 +	
260	260	260	260	110	87	26	20	61	68	0.9	4,0 +	
540	539	542	543	5			2	8.4	6.4	1.3	6,0 +	
1043	1045	1046		weak		14	~ 3				0,0 +	F–F, (F+1)–(F+1
1119	1114	1116	1116	44	56	4	~ 4	2.2	1.0	~ 2.2	2,0 +	
1172	1170	1170		230	209	^{28}Si*	5				3,3 +	F + (F + 1)
	(1195)			~ 5**								
	1224		1222	9	~ 13			29	26	1.1	3, ? −	Oct. vib.
1263	1261	1263	1262	39	~ 35	70	49	22	26	0.9	4,3 +	F + (F + 1)
	1283			26	~ 41						4,0 +	F–F, (F+1)–(F+1
	1333			~ 6	~ 12							
	1352	1349	1355	~ 8	~ 16	~ 1	~ 2	4.9	0.9			
1376	1374	~ 1373		22	~ 32	~ 2					5,3 +	F + (F + 1)
	(1415)			2								
1466	1465	1466	1465	58	46	~ 2		35	18	2.0	2,2 +	γ-vib.
	1497			~ 19	27							
1510	1507	1508		~ 8**		5	~ 2				6,3 +	F + (F + 1)
	1544			39	60							
1549		1550				26	20				3,2 +	γ-vib.
1609	1607	1604	1605	184	167	8	~ 3	4.5	3.2	1.4	2,2 +	(F + 1) − F
		~ 1634	1631				~ 2	2.2	3.5			
			1660					1.0	1.7	0.6	4,2 +	γ-vib
1663	1659	1661		154	158	42	20					
1702	1699	1702		29	~ 34	64	33				3,2 +	(F + 1) − F
			1708					15	12	1.2	(3, ? −)	(Oct. vib.)
1749	1752	1750	1747	22	~ 14	38	32	2.1	1.3			
	1775			~ 8								
1803	1803	1793	1789	19	~ 30	10	~ 6		3.3		4,2 +	(F + 1) − F
			1820					54	53	1.0	3, ? −	Oct. vib.
	1859	~ 1853		~ 2	~ 7	~ 6	~ 3					
	1886	1893		~ 2		~ 7	4					
	1925	1921		16	18	≥ 20***	12					
		1967				≥ 4***	12					
	2008			15	~ 12						1,1 +	(F + 1) − (F − 4)
			2032					21	20	1.0	3, ? −	Oct. vib.
	2046			13	~ 11						2,1 +	(F + 1) − (F − 4)
			2050					7.0	11	0.6		
		2097				4	2					
	2109			~ 18	20						3,1 +	(F + 1) − (F − 4)
		2121				3	6					
	2179	2173		6	14	40	5					
	2193			4							4,1 +	(F + 1) − (F − 4)
	2228	2218		***	10	105	38					
		2279				24	24					
		2325				46	~ 8					
		2344					19					
		2371				144	80					

* Intensity not reliable due to presence of peaks from target impurity indicated.

** Approximately 40 % of the intensity quoted is due to ^{174}Yb target impurity.

*** Intensity not reliable due to joint in photographic plates at this point.

TABLE 4. Levels in ^{174}Yb.

Prev. value ref.	Energy		$d\sigma/d\Omega$ (d,p) $\mu b/sr$		$d\sigma/d\Omega$ (d,d') $\mu b/sr$		R	$I, K\pi$	Comments
11, 12, 13)	(d,p)	(d,d')	$\theta = 60°$	$\theta = 90°$	$\theta = 90°$	$\theta = 125°$			
0	0	0	~ 0.6	~ 0.7	63000	13000	4.8	0,0 +	Gr. state rot. band
76.5	77	79	62	29	5400	3300	1.7	2,0 +	
252	251	252	38	20	71	108	0.7	4,0 +	
527		523			5.7	3	1.9	6,0 +	
1316				≤ 2				2,2 –	Oct. vib.
		1348			14				
		1380			45	39	1.2	3,2 –	Oct. vib.
~ 1520	1509			17				(6,6 +)	((F + 2) + (F + 1))
	1559			11				5,2 –	Oct. vib.
	1630	1629	73	44	38	29	1.4	2,2 +	γ-vib.
								6,2 –	Oct. vib.
	1667		24	18				(7,6 +)	((F + 2) + (F + 1))
		1696			16	19	0.8		
	1702		99	67				3,2 +	γ-vib.
	~ 1723		~ 12	~ 12					
		1760				2			
		1778			15	19	0.8		
	1799	1801	56	35	14	15	0.9	4,2 +	γ-vib.
	1841		~ 30*	~ 8					
		1846			55	55	1.0	3, ? –	Oct. vib.
	1876		~ 15	~ 15					
	1926		~ 10	24				5,2 +	γ-vib.
	1947		~ 15	35					
	2039		~ 19	10					
	2080		79	51					
	2101		~ 40	14					
	2150		~ 40	58					
	2189	2178	79	51	9	10		2,2 +	(F + 1) – (F + 4)
	2213			19					
	2237	2230	144	78		2		3,2 +	(F + 1) – (F + 4)
	2284		144	95				3,3 +	(F + 1) + (F + 4)
	2333		110	69				4,2 +	(F + 1) – (F + 4)
	2370		100	67				4,3 +	(F + 1) + (F + 4)
	2407		~ 40	11					
	2450		~ 80	47					
	2482			26				5,3 +	(F + 1) + (F + 4)

* Contains impurities?

TABLE 5. Levels in ^{176}Yb.

Energy		$\frac{d\sigma}{d\Omega}(d,d')\mu b/sr$		R	$I, K\pi$	Comments
Previous value	(d,d')	$\theta = 90°$	$\theta = 125°$			
0	0	63000	13000	4.8	0,0+	Ground state rotational band
82.1	82	4500	2700	1.7	2,0+	
270	270	75	117	0.6	4,0+	
564	565	4.4	1.8	2.4	6,0+	
1270	1254	57	27	2.2	2,2+	γ-vibration
	1340		3.2			
	1429	13	9.3	(1.4)	4,2+	γ-vibration
	1491	30	19	1.6	(3,?−)	Octupole vibration?
	1692	8.5	4.7	(1.8)		
	1767	13	12	1.1		
	1790	39	50	0.8	3,?−	Octupole vibration

tional bands by means of (d,p) and (d,t) reactions can be calculated from equations (6) if one knows the quantities $C_{jl}^2\varphi_l$ and $U(\nu)$ or $V(\nu)$. Empirical values of these parameters can be obtained from the stripping and pick-up reactions on even targets[2)]. Theoretical values could be used, although they do differ slightly from the experimental ones. For the Yb-nuclei there is good evidence that several of the single-particle states are quite pure. For such cases it might be more reasonable to use empirical values of $C_{jl}^2\varphi_l$ for predictions of the cross sections to states in even-even nuclei. This would compensate for some uncertainties in the theoretical wave functions and in the DWBA calculations. If the states in the odd nuclei are not pure, such a procedure should of course not be applied.

Table 6 shows some of the experimental values of $C_{jl}^2\varphi_l$ for several Nilsson states, obtained from (d,p) and (d,t) reactions[2)] at $\theta = 90°$. These data are normalized such that they correspond to Q-values of 4 MeV and −1.5 MeV for the (d,p) and (d,t) processes, respectively. Predictions for the intensities of the ground-state rotational bands have been made by inserting the values of $C_{jl}^2\varphi_l$ from Table 6 into equations (6). Such results are shown in columns 2, 4, 6 and 8 of Table 7 where the values presented are $(d\sigma/d\Omega)_s$. The alternate columns of Table 7 show the experimentally observed values and, thus, the ratios between values in the corresponding columns should yield the U^2 or V^2 for the particular state in the final nucleus. The last two columns of Table 8 show estimates of U^2 and V^2 obtained in this way for the four possible ground-state reactions with the two stable odd

TABLE 6. Empirical values of $C^2_{jl}\varphi_j$, $\theta = 90°$ (microbarns per steradian).

Spin	$C^2_{jl}\varphi_l(d,p), Q = 4.0$ MeV			$C^2_{jl}\varphi_l(d,t), Q = -1.5$ MeV		
j	1/2 – [521]	5/2 – [512]	1/2 – [510]	1/2 – [521]	5/2 – [512]	3/2 – [521]
1/2	75	—	4	270	—	—
3/2	7	—	124	16	—	140
5/2	29	4	64	50	12	Not obs.
7/2	40	115	26	92	270	150
9/2	5	4	6	9	7	19
11/2	4	4	Not obs.	5	8	Not obs.

targets. These are to be compared with the corresponding values in the third and fourth columns, which were obtained from studies of the transfer of neutrons into and out of the same Nilsson states in experiments with even-even targets[2]. It is seen that there is acceptable agreement between corresponding values of V^2 obtained from the different reactions. However, in both cases measured, the values of U^2 obtained from experiments with odd targets are appreciably lower than the corresponding values from experiments with even targets. As an example, one can consider the two values for the U^2 of the 1/2 – [521] orbital in ^{170}Yb (cf. Table 8) which have been determined from the ground-state transitions in the $^{171}\mathrm{Yb}(d,t)^{170}\mathrm{Yb}$ and the $^{170}\mathrm{Yb}(d,p)$ ^{171}Yb reactions. As mentioned in sect. 2, the same factor U^2 must apply to both reactions because of the principle of detailed balance. Although the (d,p) and (d,t) reactions are not exactly inverse, the use of the empirical values of $C^2_{jl}\varphi_l$ in the cross section comparisons should ensure quite close agreement between the reduction factors U^2, and the observed discrepancy of more than a factor of two is therefore surprising. It should, however, be remembered that the ground-state Q-values for the (d,t) reactions on odd targets are approximately 1.5 MeV higher than those on even targets. The predicted experimental cross sections therefore depend on the Q-dependence obtained from the DWBA calculations (Fig. 11 of ref. 2). More complete calculations with improved triton potentials[23] give a Q-dependence less steep than the one used here. An estimate shows that the values for U^2 in Table 8 obtained from odd targets consequently should be increased by approximately 25 $^0/_0$. This correction is, however, not sufficient to remove the discrepancy with the values obtained from even targets, and the origin of this discrepancy is therefore still unknown. In order to be consistent with the earlier work, no correction has been applied to the values in Tables 7 and 8.

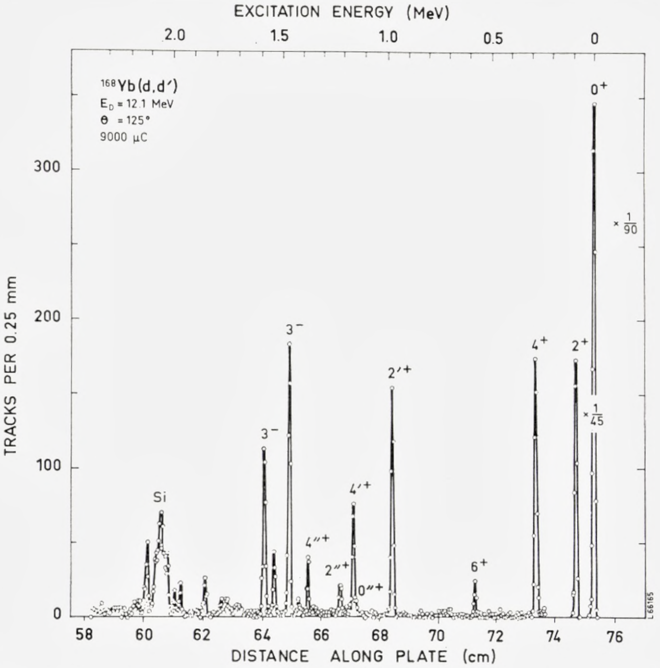

Figure 5. Deuteron spectrum for the reaction ^{168}Yb (d,d') at $\theta = 125°$.

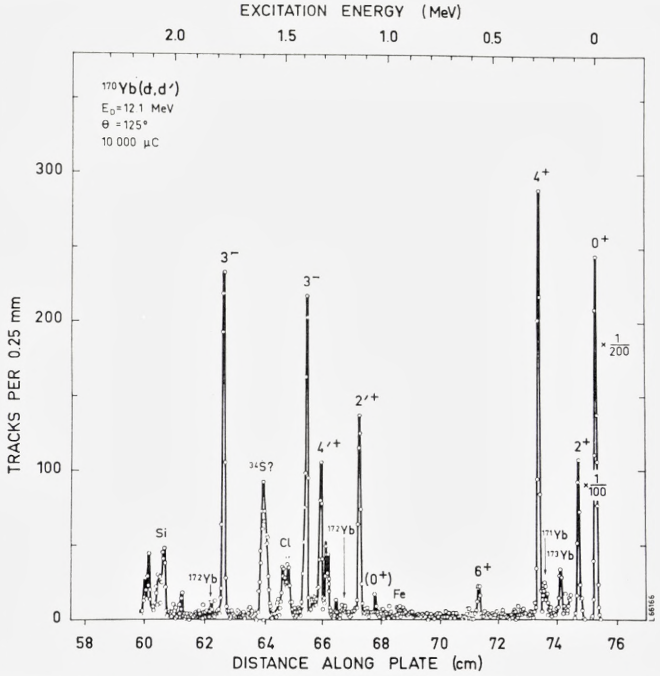

Figure 6. Deuteron spectrum for the reaction ^{170}Yb (d,d') at $\theta = 125°$.

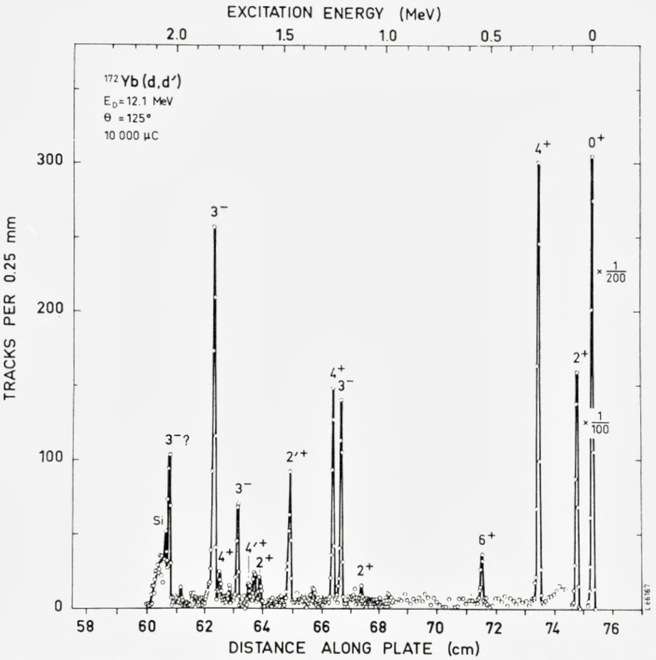

Figure 7. Deuteron spectrum for the reaction ^{172}Yb (d,d') at $\theta = 125°$.

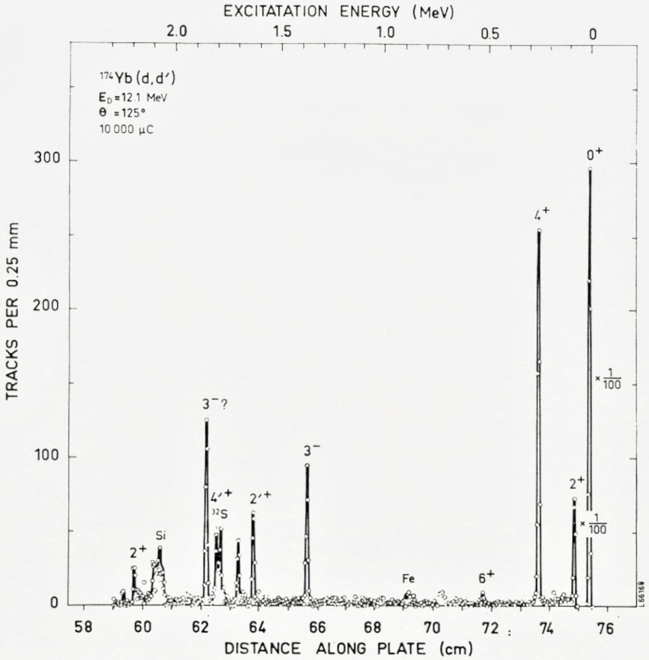

Figure 8. Deuteron spectrum for the reaction ^{174}Yb (d,d') at $\theta = 125°$.

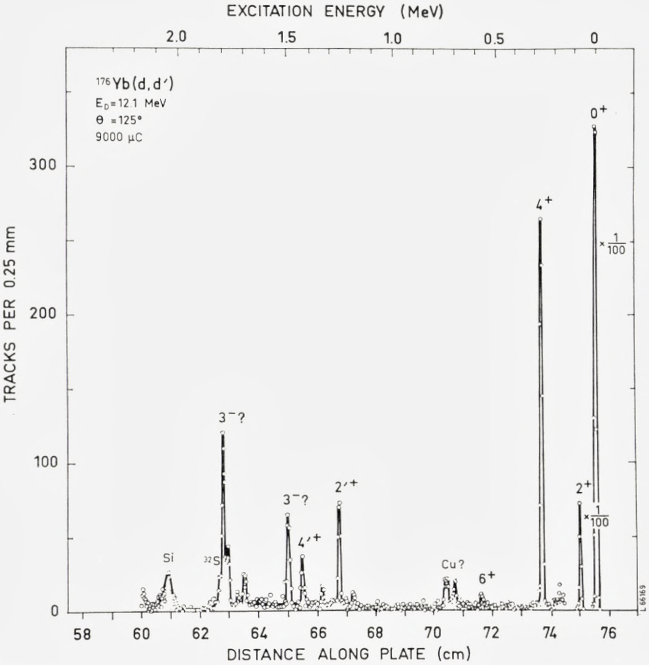

Figure 9. Deuteron spectrum for the reaction ^{176}Yb (d,d') at $\theta = 125°$.

4.2. Quadrupole γ-vibrations

The $K = 2$ quadrupole vibrational states in deformed rare earth nuclei have been the subject of many investigations. There are good theoretical predictions of the observed systematics of excitation energies and transition probabilities to the ground states$^{17, 18)}$. Due to the nature of the states near the Fermi surface in the ytterbium region, the γ-vibrations occur at rather high excitation energies and the B(E2) values connecting them with the ground states are small$^{17)}$. As a result, most of these states were not observed in a recent survey of vibrational states in rare earth nuclei by means of the Coulomb excitation process$^{19)}$. The energies of the γ-vibrational states in ^{168}Yb, ^{170}Yb, ^{172}Yb, and ^{176}Yb were previously known$^{7, 17, 11)}$ although there has been some uncertainty concerning the assignment of the 1.468 MeV state in ^{172}Yb as a γ-vibrational state$^{20)}$.

It has been shown that the collective vibrational states in the even nuclei of samarium and gadolinium were strongly excited by the (d,d') process. The quadrupole excitations could with reasonable certainty be identified on

Table 7. Comparison of ground state rotational band populations with predictions $\theta = 90^\circ$.

Spin	Transfer of 1/2 – [521] neutron				Transfer of 5/2 – [512] neutron			
	^{171}Yb (d,t) ^{170}Yb		^{171}Yb (d,p) ^{172}Yb		^{173}Yb (d,t) ^{172}Yb		^{173}Yb (d,p) ^{174}Yb	
	Predicted $\left(\frac{d\sigma}{d\Omega}\right)_s$	Exp. $\frac{d\sigma}{d\Omega}$	Predicted $\left(\frac{d\sigma}{d\Omega}\right)_s$	Exp. $\frac{d\sigma}{d\Omega}$	Predicted $\left(\frac{d\sigma}{d\Omega}\right)_s$	Exp. $\frac{d\sigma}{d\Omega}$	Predicted $\left(\frac{d\sigma}{d\Omega}\right)_s$	Exp. $\frac{d\sigma}{d\Omega}$
0	520	130	36	36	8.5	4	0.8	0.7
2	118	30	20	16	344	136	45.4	29
4	162	47	27	20	213	87	34.0	20
6	7	5	3	2	10	Not obs.	3.5	Not obs.

the basis of the ratio of the yields at two angles. Furthermore, the inelastic scattering cross section at 90° was, within the experimental error, proportional to the reduced transition probability B(E2).

The inelastic scattering spectra shown in Figs. 5–9 contain several strong groups for which the ratio, R, of the 90° yield to that at 125° is in the range 2.0—3.0, which is typical for quadrupole excitations. In each spectrum, one such group was found which could be ascribed to the $K = 2$ γ-vibrational state. The γ-vibrational bands seem to be characterized by fairly strong populations of the 2+ and 4+ states, whereas the 3+ states, which have unnatural parity, were never observed with certainty.

The above mentioned properties definitely establish the 1468 keV state in ^{172}Yb as the γ-vibrational 2+ state. A similar state was observed at 1630 keV in ^{174}Yb.

Some of the properties of the γ-vibrational states have been collected in Table 9. The reduced transition probabilities, B(E2), given there are based

Table 8. Experimental values of U^2 and V^2.

Nilsson state	Ground state of isotope	From experiments with even-even targets²		From experiments with odd targets	
		U^2 (%)	V^2 (%)	U^2 (%)	V^2 (%)
1/2 – [521]...........	^{170}Yb	54 ± 8	44 ± 7	26 ± 4	–
1/2 – [521]...........	^{172}Yb	30 ± 4	74 ± 11	–	86 ± 13
5/2 – [512]...........	^{172}Yb	74 ± 11	28 ± 4	40 ± 6	–
5/2 – [512]...........	^{174}Yb	27 ± 4	75 ± 11	–	62 ± 9

TABLE 9. Properties of $K = 2+$ gamma-vibrational states.

Isotope	E_{2+}	$\hbar^2/2\mathfrak{J}$	B(E2)[a)]	$\mathrm{B(E2)}_{\mathrm{s.p.u.}}$[b)]
^{168}Yb	981	13.4	0.15	5.2
^{170}Yb	1145	13.2	0.094	3.2
^{172}Yb	1465	13.9	0.044	1.5
^{174}Yb	1629	12.2	0.050	1.7
^{176}Yb	1254	12.5	0.070	2.4

a) $\mathrm{B(E2)}e^2$ in units of 10^{-48} cm^4 $= 1.18 \times (d\sigma/d\Omega)_{90°}$ in *mb*/*sr*.
b) $1\mathrm{B(E2)}_{\mathrm{s.p.u.}} = 0.029 \times 10^{-48} e^2 \mathrm{cm}^4$.

upon the semi-empirical fact that the inelastic cross sections are proportional to the reduced transition probability between the ground state and the excited state. The proportionality constant was obtained by interpolation from (d,d') data for the even isotopes of Sm, Gd, Th and U. Furthermore, for ^{176}Yb, the B(E2) value was already known[19)]. The value obtained here is in good agreement with the earlier value. The 90° (d,d') cross sections were used for the evaluation of the B(E2) values because it was felt that they were less affected by multiple excitations involving the 4+ states.

Fig. 15 shows some of the systematic features of the γ-vibrational states as functions of the mass number. For the nuclei ^{172}Yb and ^{174}Yb the energies are high and the transition probabilities low. Thus, for these nuclei, typical collective properties are less well developed than for other nuclei in the rare earth region.

Additional information concerning these levels is obtained from the (d,p) and (d,t) reactions in the ^{170}Yb, ^{172}Yb, and ^{174}Yb nuclei. Current theories consider the collective vibrations to consist of superpositions of quasiparticle states and the decomposition of the $K = 2$ γ-vibrations has been calculated[17), 18)]. These two-particle states can be populated in the stripping and pick-up reactions by coupling the transferred neutron (or hole) with the unpaired nucleon in the odd mass target. There is an obvious selection rule that one can populate only the two-particle states for which one of the particles is the ground state of the odd target nucleus.

In most cases, the calculations show that only one or two components of the vibrational states have appreciable amplitude[17)] and also satisfy the above selection rule. Thus, it is easy to calculate the predicted intensities for the population of a vibrational state by a (d,p) or (d,t) reaction from its theoretical composition. However, it should be remembered that, when more than one component is present, the amplitudes θ_{jl} in the expression for the spectroscopic factor add coherently—that is, they must be added for

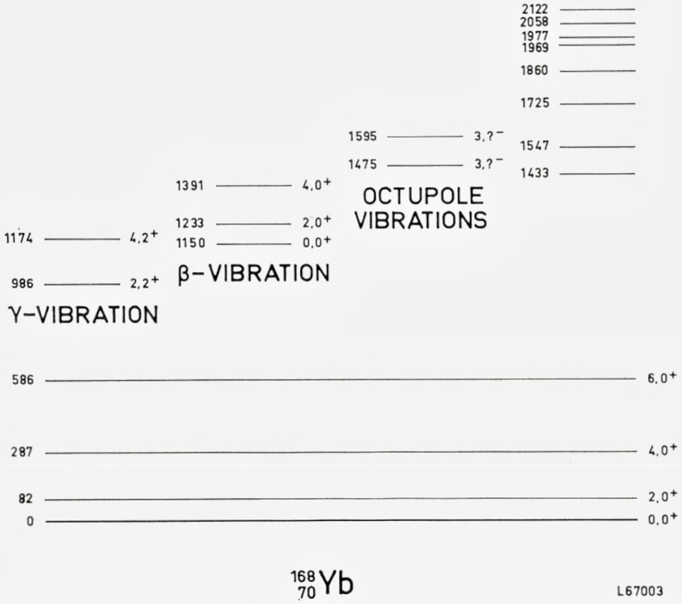

Figure 10. Level scheme for ^{168}Yb. The numbers on the left side of each level indicate the excitation energy in keV, and those on the right are the quantum numbers $I, K\pi$.

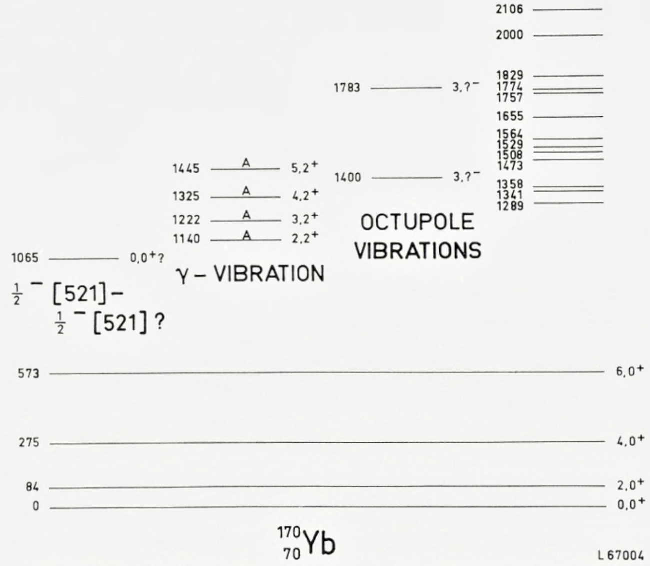

Figure 11. Level scheme for ^{170}Yb. See caption to Figure 10.

a given j before being squared. Only the magnitude of $C_{jl}^2\varphi_l$ and not its sign was obtained from the experiments with even-even targets. Therefore, in all calculations of mixed states which follow, each term has been given the same sign as the theoretical value of C_{jl}.

The $1/2-[521]$ Nilsson orbital forms the ground state of ^{171}Yb. For the γ-vibrational state in ^{170}Yb, Bès et al.[17] predict two components which should be populated in the (d,t) reaction. These are the $1/2-[521]+3/2-[521]$ state with amplitude 0.45 and the $1/2-[521]-5/2-[523]$ state with amplitude 0.52. Empirical values of $C_{jl}^2\varphi_l$ for the $3/2-[521]$ and $5/2-[523]$ states from (d,t) experiments on even Yb targets[2] then predict the cross sections[*] for the 2+, 3+, 4+, and 5+ members of the $K = 2$ γ-vibration to be 33, 14, 13, and 10 $\mu b/sr$, respectively. The experimental cross sections for these states are 45, 14, 32, and ~3 $\mu b/sr$. As we shall see later, there is a good possibility that the experimental value quoted for the 2+ member actually is that for a closely spaced doublet, the other state having a cross section of ~5 $\mu b/sr$. This would reduce the observed population of the 2+ state to ~40 $\mu b/sr$. Similarly, the cross section of the 4+ state may be reduced by ~8 $\mu b/sr$ to ~24 $\mu b/sr$.

The agreement between prediction and experiment is not very good for the 4+ and 5+ states. One might try to account for the high cross section of the 4+ state by considering the possibility that the observed value pertains to additional unresolved levels. Of course, this line of argument cannot be applied to the 5+ level where the observed strength is only about 3 $\mu b/sr$ compared to a predicted value of 10 $\mu b/sr$. It is possible, however, to calculate which set of amplitudes, if any, for the above mentioned two-quasiparticle components would yield the experimental cross sections. One can calculate, first, the effect of varying the relative amplitudes of the two components to reproduce the relative intensities of the different members of the rotational band and, then, normalize both amplitudes to obtain agreement with the absolute values of the observed cross sections. Let I_2, I_3, I_4, and I_5 be the intensities of the 2+, 3+, 4+, and 5+ states, respectively, and let "a" and "b" be the respective amplitudes of the $1/2-[521]+3/2-[521]$ and $1/2-[521]-5/2-[523]$ components. In Fig. 16, the solid curves show the predicted intensity ratios I_2/I_3, I_4/I_3, and I_5/I_3 as a function of the ratio b/a. The cross-hatched areas show the experimentally observed relative intensities. It is seen that the predicted and the experimental values of I_2/I_3 are in good agreement if b/a is less than 0.8, and for I_4/I_3 the value of b/a must be less than about 0.6. The experimental ratio I_5/I_3 places a still lower

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[*] All comparisons of cross sections in the present work are based on the 90° data.

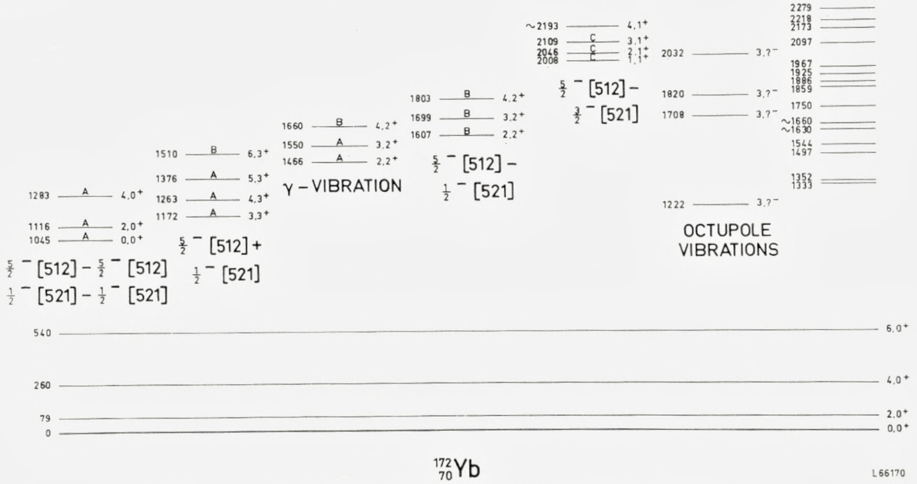

Figure 12. Level scheme for ^{172}Yb. See caption to Figure 10.

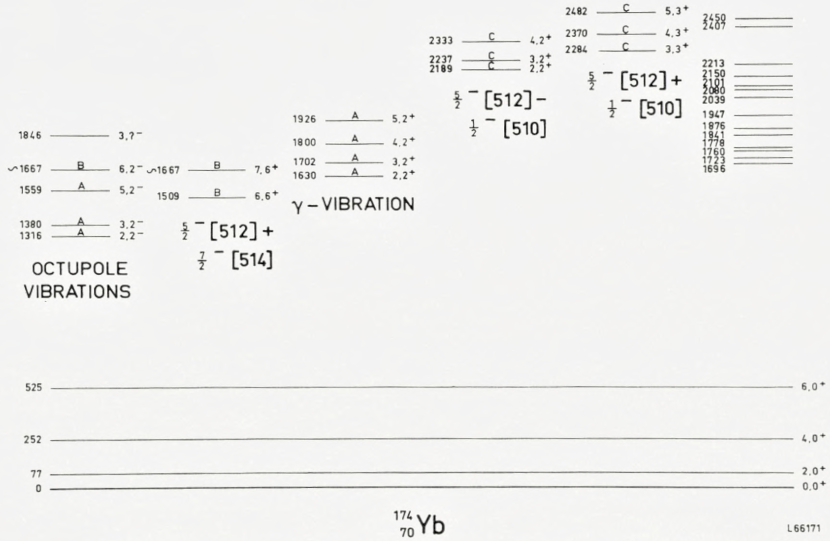

Figure 13. Level scheme for ^{174}Yb. See caption to Figure 10.

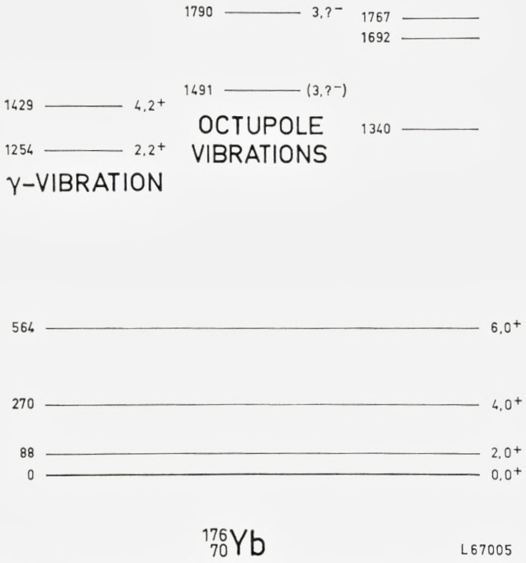

Figure 14. Level scheme for ^{176}Yb. See caption to Figure 10.

limit on b/a. The relative uncertainty in I_5 is large as the cross section is weak, but even the generous relative error of 100 $^0/_0$ would limit the ratio b/a to less than 0.4. The absolute values of the cross sections give $a \approx 0.53$ and $0.05 \lesssim b \lesssim 0.2$. The amplitude of the $1/2-[521]+3/2-[521]$ component is seen to be in good agreement with the theoretical value of 0.45, but that of the $1/2-[521]-5/2-[523]$ component is less than about one half of the theoretical value 0.52.

In ^{172}Yb, the only component of the 1.468 MeV, $K = 2$, γ-vibration which can be populated in either the (d,p) or the (d,t) reaction is a fragment of the $1/2-[521]-5/2-[512]$ state. According to Bès et al.[17], this component should have an amplitude of 0.20. On the other hand, the calculations of Soloviev[18] suggest that the state at 1.468 MeV is almost a pure $1/2-[521]$ $-5/2[512]$, $K = 2$ two-quasiparticle state. From the (d,d') systematics discussed above it is seen that most likely the band is the $K = 2$ γ-vibration. The stripping and pick-up reactions can provide an empirical estimate of the amplitude of the above two-quasiparticle configuration which forms a part of this vibration. As this band was not strongly populated in either the (d,p) or (d,t) process, the weaker peaks were not observed. Hence, in this discussion, only the member of the band with the largest cross section is considered. In the (d,t) reaction the $2+$ member of the band is expected

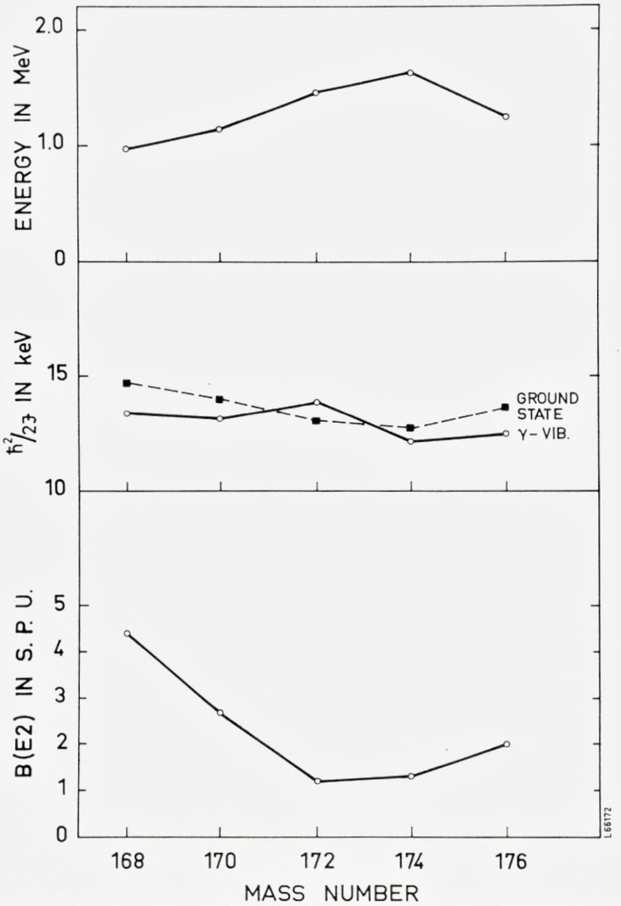

Figure 15. Properties of γ-vibrational states in even Yb nuclei. The top graph shows the energy of the $I,K\pi = 2,2+$ state. The centre figure shows the inertial parameter $\hbar^2/2\mathfrak{J}$ for the rotational bands based on the γ-vibrations (solid lines) compared with values for the ground state rotational bands (dashed lines). In the bottom graph are found the reduced transition probabilities which connect the γ-vibrational states to the ground states.

to be populated most strongly. For the pure two-quasiparticle configuration, this state would be expected to have a cross section of 185 $\mu b/sr$ if one assumed a reasonable value of $V^2 \approx 0.8$ for the $1/2-[521]$ orbital in ^{173}Yb. This estimate of V^2 was obtained from an interpolation of the results of ref. 2. The experimental cross section for the $2+$ state at 1.468 MeV is 46 $\mu b/sr$, which implies that the state is about 25 $^0/_0$ $1/2-[521]-5/2-[512]$. An independent estimate of the amplitude of this component can be obtained from the (d,p) spectra where the $3+$ member of the band is the most strongly populated. Again using the values of $C_{jl}^2\varphi_l$ of Table 6, and $U^2 \approx 0.8$ for the $5/2-[512]$ orbital in ^{171}Yb, the pure two-quasiparticle state would be expected to have a cross section of 62 $\mu b/sr$. The experimental cross section is 20 $\mu b/sr$ so that the vibrational state contains $\sim 32\ ^0/_0$ of the two-quasiparticle state. The two estimates of the admixture are consistent, within experimental error, and indicate that the amplitude for the $1/2-[521]-5/2-[512]$ component of the $K = 2$ γ-vibration in ^{172}Yb is about 0.50 or 0.55 as compared to the theoretical estimate of 0.20[17)].

In ^{174}Yb, there is again only one two-quasiparticle state which is predicted to have a large component in the γ-vibration. This is the $5/2-[512]-1/2-[510]$ state with a theoretical amplitude of 0.56[17)]. The $1/2-[521]-5/2-[512]$ state is also expected to contribute an amplitude of 0.07, but, coupled with a low value of U^2 for the $1/2-[521]$ state in ^{173}Yb, this is found to result in a negligible correction to the cross sections. The $2+$, $3+$, $4+$, and $5+$ members of the rotational band based on the $K = 2$ γ-vibration are then expected to have (d,p) cross sections of 27, 32, 17, and 6 $\mu b/sr$, respectively. The experimental values are 44, 67, 35, and < 22 $\mu b/sr$. It is seen that the observed cross sections are almost twice as large as those predicted, which would suggest that the amplitude of the $1/2-]510]-5/2-[512]$ state is closer to 0.75 or 0.80 than the theoretical value of 0.56. It is also noted that the predicted ratios of the cross sections to the $2+$, $3+$, $4+$, and $5+$ states are 0.85:1.00:0.53:0.19 as compared with experimental values of 0.66:1.00:0.52:<0.33, respectively. These ratios agree reasonably well, although the intensity of the $2+$ state is a little weaker than expected, compared to the other members of the band. However, if the amplitude of the $1/2-[521]-5/2-[512]$ state were ~ 0.5 instead of 0.07, the predicted intensity ratios would be in agreement with the experimental data.

4.3. Octupole Vibrations

Just as in the case of the quadrupole excitations discussed in the previous section, the collective octupole vibrations are strongly excited by the (d,d') process. The ratios R of the 90° yield to the 125° yield for the Sm and Gd nuclei were found to be in the range of 1.2—1.6, with some tendency toward a decrease with increasing mass numbers.

The inelastic scattering spectra shown in Figs. 5—9 contain several strong deuteron groups with R in the range of 1.0—1.3. Although the absolute intensities of these groups are considerably smaller than those observed in

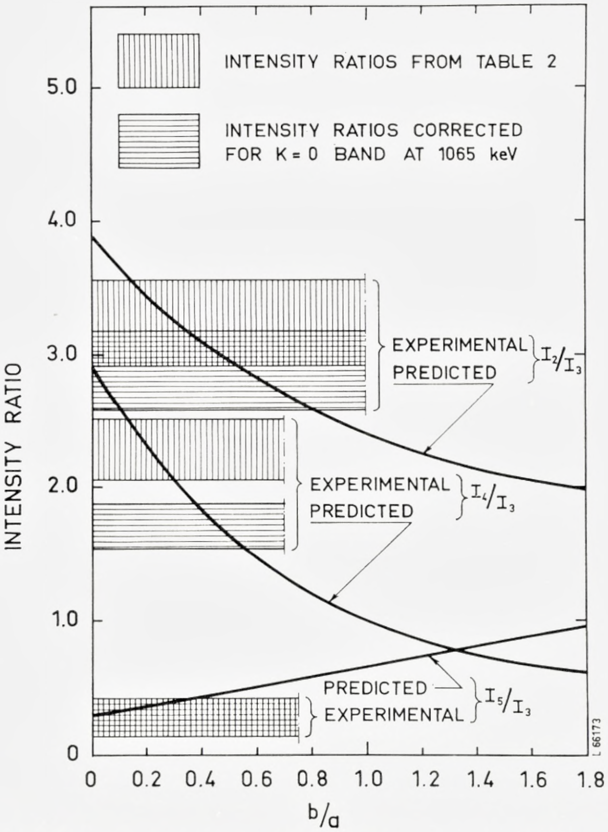

Figure 16. Intensity ratios for members of the γ-vibrational band in ^{170}Yb populated by the (d,t) reaction. The letters a and b are the amplitudes of the 1/2 – [521] + 3/2 – [521] and 5/2 – [523] – 1/2 – [521] components, respectively. The experimental values shown by vertical cross-hatching were obtained with the assumption that no other unresolved states contribute to the observed cross sections. However, it is possible that the 2 + and 4 + states are actually unresolved doublets (see text), in which case the present interpretation indicates that the adjusted intensity ratios shown by horizontal cross-hatching are applicable.

the lighter deformed rare earth nuclei, it is natural to associate at least some of them with excitations of collective octupole states. Some of the properties of states believed to be of this nature are collected in Table 10.

On the basis of the (d, d') results it has been possible to identify the spin 1, 3, and 5 members of rotational bands based on $K\pi = 0-$ octupole vibrations in the Sm and Gd nuclei[16, 21]. In ^{152}Sm, these rotational members have also been found in a vibrational band known to be $K\pi = 1-$. The absence of any deuteron groups which can be assigned to possible spin 1 members

TABLE 10. Properties of octupole vibrational states

Isotope	E_{3-}	K	$\hbar^2/2\mathfrak{J}$ keV	B(E3)[b)]	$B(E3)_{s.p.u.}$[c)]
^{168}Yb	1475			0.077	6.2
–	1595			0.039	3.1
^{170}Yb	1400			0.049	3.9
–	(1783)[a)]			(0.045)	(3.6)
^{172}Yb	1222	(2)		0.026	2.1
–	(1708)			(0.015)	(1.2)
–	1820			0.053	4.2
–	2032			0.022	1.8
^{174}Yb	1380	2	10.0	0.041	3.3
–	1846			0.051	4.1
^{176}Yb	1491			0.027	2.2
–	(1790)			(0.038)	(3.0)

a) Brackets indicate that the assignment is doubtful.
b) $B(E3)/e^2$ in units of $10^{-72}cm^6 = 0.85\,(d\sigma/d\Omega)_{90^\circ}$ in *mb/sr*.
c) $1\,B(E3)_{s.p.u.} = 1.25 \times 10^{-74} e^2 cm^6$.

associated with most of the lower levels in Table 10 suggests that these are not members of the $K = 0$ or $K = 1$ bands. From the following discussion it will be seen that there is evidence supporting the assignment of $K = 2$ to one of these states. The appearance of $K\pi = 2-$ states at the lowest energy is also in agreement with the fact that the lowest lying octupole state in ^{166}Er has recently [22, 23, 24)] been established to have a K-value of 2.

In Table 10, columns 5 and 6 contain B(E3) values for these octupole states. These values were obtained from the (d, d') intensities in a manner similar to that described for the quadrupole excitations. It is believed that this procedure will yield B(E3) values within an uncertainty of 40 %. A slight correction for Q-dependence of the proportionality constant was obtained from a DWBA calculation.

The largest E3 transition probabilities found in the Yb nuclei are approximately five single-particle units. This number can be compared to values of 20 to 30 single-particle units for nuclei at the beginning of the deformed rare earth region. Thus, there is a very considerable decrease in the strength of the octupole levels below approximately 2 MeV as one increases in mass from the Sm and Gd regions.

As in the case of γ-vibrations, the single-particle transfer reaction can sometimes provide important additional information. The lowest $3-$ state in ^{170}Yb is at 1400 keV. A peak corresponding to this energy is also observed in the (d,t) spectrum. Of the 19 $\mu b/sr$ observed approximately 7 $\mu b/sr$ are ascribed to the ^{172}Yb impurity in the target. The peaks in the triton spectra at 1289, 1341, and 1358 keV might possibly be associated with lower spin members of an octupole rotational band. In the (d,d') spectrum, there is a multiple group at ~1300 keV, part of which might correspond to a 1 – state. If this is indeed the case, it is impossible on the basis of the present data to decide whether this band has $K = 0$ or $K = 1$. Both of these possibities would involve small amplitudes of states[18)] found by the transfer of the 1/2 + [400] or the 3/2 + [402] neutrons and are consistent with the weak triton groups observed.

For ^{172}Yb, the lowest energy 3 – state seen in the (d,d') process is at 1222 keV. This level is also weakly populated in the (d,t) reaction, but as in the case of ^{170}Yb, the data do not give a unique determination of the K-value. Any possible band members with spin less than 3 could be weakly populated or could be obscured by the large triton groups at 1119 and 1172 keV.

The wave functions for the octupole states in this region are likely to contain components involving the 1/2 + [400] and 3/2 + [402] Nilsson orbitals[18)] which have (d,t) cross sections of the order of ~100 $\mu b/sr$ for the population of a spin 3 state at this Q-value. The observed cross section of ~13 $\mu b/sr$ thus gives an upper limit of 10–15 % for the admixture of either of these states into the octupole vibration.

The state at 1222 keV is not observed in the (d,p) reaction, which is consistent with the fact that there are no nearby Nilsson states above the Fermi surface connected to the 1/2 – [521] orbital with large octupole matrix elements.

As seen from Table 10, there are three other states in ^{172}Yb which have large (d,d') cross sections, and R values which would suggest they are octupole excitations. The state at 1707 keV coincides, within the experimental error, with a state assigned as $I, K\pi = 3,2+$ (see section 4.6). It is highly improbable that this is the same state as that observed in the (d,d') spectrum, because experience has shown that the latter process populates only natural parity states. The remaining 3 – states have no corresponding peaks in the (d,p) or (d,t) spectra with cross sections greater than ~5 $\mu b/sr$.

The lower energy 3 – state observed in the (d,d') spectrum of ^{174}Yb is at 1380 keV. It is interesting to speculate that this might be one member of a $K = 2$ band based on the 2 – state at 1320 keV. This 2 – level was previously assigned to be the 9/2 + [624] – 5/2 – [512] two-neutron configuration[11)], which is consistent with the present suggestion if one considers that the $9/2+[624]-5/2-[512]$ combination is predicted[18] to make up a large fraction of the $K\pi = 2-$ octupole vibration. Further evidence for this assignment is obtained from the results of the (d,p) reaction in which this two-quasiparticle component can be populated by the transfer of a $9/2+[624]$ neutron. The energies given above for the spin 2 and 3 members indicate that the inertial parameter $\hbar^2/2\mathfrak{J}$ is 10 keV, which suggests the 4, 5, and 6 spin members should be at approximately 1460, 1560, and 1680 keV. The expected cross sections for the spin 2, 3, 4, 5, 6, 7, 8, and 9 spin members of the rotational band are 2, ~ 1, 8, 11, 8, 3, ~ 1 and ~ 0 $\mu b/sr$, respectively. Thus, the proton groups corresponding to the spin 2 and 3 members should hardly be observable and would be lost in the small background seen in these regions of the spectrum. The spin 4 state is unfortunately obscured by protons from the ^{13}C impurity in Fig. 4. However, the group at 1559 keV with ~ 11 $\mu b/sr$ can be ascribed to the spin 5 member, and part of the 18 $\mu b/sr$ in the peak corresponding to 1667 keV can be due to the $6-$ state. (The remainder of the 1667 keV group will be explained later). Thus it is seen that all the available data are consistent with the possibility that the $K = 2$ octupole vibration exists at 1320 keV and that this collective state contains a large admixture of the $9/2+[624]-5/2-[512]$ two-neutron configuration.

4.4. Low-Lying $K = 0$ Bands

It is known that a number of low-lying bands with $K\pi = 0+$ exist in deformed rare earth nuclei. In particular, low-lying collective states which are connected to the ground-state band with fairly large electric quadrupole matrix elements have been found in some cases. These states are often called β-vibrations, although their character is not entirely clear. In addition, several other $K = 0$ bands have been identified at excitation energies less than 2 MeV. The present experiments add to our knowledge of the nature of these states in two ways. Firstly, the transfer reactions enable us to determine the amplitudes of certain two-quasiparticle components in the state, and secondly, the excitation by (d,d') of the $2+$ states allows an estimate of the reduced transition probabilities to the ground state.

As for the ground-state bands, special attention is required when the transferred neutron is in the same Nilsson orbital as the unpaired target nucleon. It has been seen that a state filled with a pair of nucleons $(\nu,\bar{\nu})$ is present in the correlated ground state of a nucleus with the probability $V^2(\nu)$. Hence, there must be a probability $1-V^2(\nu)$ or $U^2(\nu)$ that this state is distributed among $K = 0+$ excited states in the spectrum. One would expect that these $K = 0$ bands in the (d, t) and (d, p) processes are populated in the same way as the ground-state rotational bands. For instance,with a (d, t) reaction on a ^{173}Yb target one could remove the single $5/2-[512]$ neutron. This process should take place with a cross section $\left(\frac{d\sigma}{d\Omega}\right)_s$, but it has been seen that only a fraction U^2 of this strength leads to the ground state of ^{172}Yb. Therefore, a cross section $V^2\left(\frac{d\sigma}{d\Omega}\right)_s$ must lead to the population of other $K = 0+$ states which have admixtures of the $5/2-[512]-5/2-[512]$ configuration. This can also be shown by considering the wave function of the component of state $(\nu, \bar{\nu})$ which is orthogonal to the ground state (i.e., is in excited states) and is[6)]

$$|v = 2,\ (\nu,\bar{\nu})\rangle = \left\{V(\nu) - U(\nu)a^{+}(\nu)a^{+}(\bar{\nu})\right\} \prod_{\nu' \neq \nu} \left\{U(\nu)' + V(\nu')a^{+}(\nu')a^{+}(\bar{\nu}')\right\}|0\rangle. \tag{7}$$

The probability of populating this component in, say, a (d, p) reaction from an odd target nucleus in a state $\alpha^{+}(\bar{\nu})\,|\,v = 0\rangle$ is

$$|\langle v = 2, (\nu,\bar{\nu})\,|\,a^{+}(\bar{\nu})\alpha^{+}(\nu)\,|\,v = 0\rangle|^2\left(\frac{d\sigma}{d\Omega}\right)_s = U^2(\nu)\left(\frac{d\sigma}{d\Omega}\right)_s. \tag{8}$$

Similarly, the (d, t) cross section to the portion of the state $(\nu, \bar{\nu})$ orthogonal to the ground state is

$$|\langle v = 2, (\nu,\bar{\nu})\,|\,a(\nu)\alpha^{+}(\nu)\,|\,v = 0\rangle|^2\left(\frac{d\sigma}{d\Omega}\right)_s = V^2(\nu)\left(\frac{d\sigma}{d\Omega}\right)_s. \tag{9}$$

In these expressions, $U^2(\nu)$ and $V^2(\nu)$ pertain to the filling of the ground state of the final nucleus. Of course, the relative intensities for the populations of the various members of the rotational bands based on these states are expected to be the same as those in the ground-state band in any one reaction.

In the (d, p) and (d, t) spectra for reactions leading to ^{172}Yb, one sees particle groups corresponding to states at excitation energies of 1045, 1115, and 1283 keV in ^{172}Yb. The energy spacings of these levels are reasonable for the $0+$, $2+$, and $4+$ members of a $K = 0+$ band with a slightly greater moment of inertia than that of the ground state. It is also seen that, in both processes, the relative intensities for the rotational members are strikingly similar to those observed for the ground-state band. This is particularly noticeable for the 1045 keV level which should be populated by $l = 1$

TABLE 11. Comparison of (d,p) and (d,t) populations of the $K = 3$ band at 1172 keV in ^{172}Yb. Intensities shown are cross sections in $\mu b/sr$ at $\theta = 90°$.

Spin	Predicted intensities for 1/2 – [521] + 5/2 – [512] state		Predicted intensities for 1/2 – [521] – 7/2 – [514] state		Experimental intensities	
	(d,t)	(d,p)	(d,t)	(d,p)	(d,t)	(d,p)
3	270	11	0	6	209	5
4	54	66	0	15	35	49
5	42	3	0	4	32	< 2
6	~ 13	1	0	0.1	~ 5	~ 2

stripping in the (d,p) process. In order to enhance the intensity of the proton group corresponding to this state, an exposure was made at $\theta = 45°$ where a maximum in the angular distribution for $l = 1$ stripping is expected for the Q-value and beam energy used in this work[25]. It is seen that the 1045 keV state is populated quite strongly in this spectrum. This test was considered as an important verification of the existence and nature of the state, as the 1045 keV level itself is populated only very weakly in the (d,t) process.

It is interesting to note that, except for the ground-state rotational band, no excited states below 1174 keV have previously been reported in ^{172}Yb. The present assignment of $K\pi = 0+$ gives a good explanation. A $K = 0$ band would not be expected to be strongly populated in the electron capture decay of ^{172}Lu which has $K\pi = 4-$. In the β^--decay of ^{172}Tm, which has $K\pi = 2-$, the transitions to the 1045 keV band would have much smaller Q-values than those leading to the ground-state band and could easily have been missed. However, a careful study should reveal the β^--feeding of these levels and, in fact, the 1044 and 1119 keV states have recently been observed in a study of the β-decay of ^{172}Tm, using lithium-drifted germanium γ-ray detectors[10].

On the basis of the absolute values of the cross sections, it is possible to make some comments about the 1/2 – [521] – 1/2[521] and 5/2[512] – [512] components of the 1045 keV state. If all the components of the wave function orthogonal to the ground state appear in one excited band, the above expressions give the expected cross sections for levels in that band. If only a fraction of this component appears in a given $K = 0$ excited band, the cross sections will be correspondingly reduced.

In the reaction ^{173}Yb$(d,t)^{172}$Yb, the population of the band based on the 1045 keV level is ~30 % of the value $\left(\frac{d\sigma}{d\Omega}\right)_s$ calculated from the data in Table 6. The value of V^2 for the 5/2 – [512] orbital in the ground state of ^{172}Yb is ~30 % according to Table 8. Thus, it seems that most of the 5/2 – [512] – 5/2 – [512] amplitude which is not in the ground state is found in the band based on the 1045 keV state.

For the (d,p) reaction, the cross section to the 1045 keV level at $\theta = 90°$ is too weak to be reliable. However, one can make use of the data at $\theta = 45°$. There, the cross section for the 1045 keV level is ~13 % of that for the ground state. This value can be used to estimate that the cross section for the 1045 keV level is $0.09\left(\frac{d\sigma}{d\Omega}\right)_s$. Thus, the value of U^2 for the 1/2 – [521] orbital in the ground state of ^{172}Yb is at least 0.09. This is also consistent with the data given in Table 8.

A state with a similar nature may also have been observed in ^{170}Yb at an excitation energy of 1065 keV. This assignment is not nearly as certain as the one in ^{172}Yb, but it does give a good explanation of the (d,t) data. Although complete angular distributions have not been measured, the variation of the (d,t) cross section for the 1065 keV state over the four angles measured closely resembles that for other states populated by $l = 1$ pick-up. This aspect of the reaction has not been well studied, but it is possible to say that this state is not populated by a pick-up process with high angular momentum transfer. This excludes the possibility that the 1065 keV level could be one of the four possible two-quasiparticle states formed by picking up a 7/2 + [633] or 5/2 – [523] neutron, as these states are populated predominantly by processes involving the transfer of 4, 5, or 6 units of angular momentum. Furthermore, the spectrum does not show a rotational band based on the 1065 keV state with an intensity pattern consistent with pick up of a 3/2 – [521] neutron to form a $K = 1$ band. If this band had the same relative intensities as the ground-state band, the 2+ and 4+ states would be expected to have cross sections of 5 and 8 $\mu b/sr$, respectively. These states would be at almost the same excitation energies as the 2+ and 4+ members, respectively, of the γ-vibrational band discussed earlier and give rise to the estimated ~5 $\mu b/sr$ and 8 $\mu b/sr$ which might have to be subtracted from the cross sections to those states.

The (d,t) cross section to the 1065 keV state is ~8 % of the value $\left(\frac{d\sigma}{d\Omega}\right)_s$ expected for a pure 1/2 – [521] – 1/2 – [521] state. This means that V^2 for the 1/2 – [521] orbital in the ground state of ^{170}Yb is at least 0.08. From experiments with even targets it appears that the V^2 is actually about 0.44.

Thus, a large component of this $K = 0$ configuration is yet to be assigned or some other hindrance occurs.

There is empirical evidence from the lighter rare earth nuclei that the (d,d') reaction populates the $0+$, $2+$, and $4+$ members of excited $K = 0$ bands[15, 16]. The cross sections for these states are approximately equal at 125°, but at 90° the $2+$ state is usually dominant.

The (d,d') spectrum obtained for ^{168}Yb (Fig. 5) gives evidence for the population of a $K = 0$ band based at a level at 1150 keV. This band has been observed earlier[7]. The estimated B(E2) value for the $2+$ state is given in Table 9.

It will be difficult to observe the $2+$ and $4+$ members of the suggested $K = 0$ band in ^{170}Yb with the (d,d') process, because these states occur at almost the same energies as strongly populated members in the γ-vibrational band. In the 125° spectrum, there is weak evidence for a peak corresponding to the $0+$ state.

The (d,d') spectra for ^{172}Yb show a small population of the $2+$ member of the $K = 0$ band based on the 1045 keV state discussed above. The cross section is an order of magnitude smaller than for the ^{168}Yb case, which indicates that this band has a different nature than those usually classified as β-vibrations.

4.5. Other Levels in ^{170}Yb

There are a number of states below an excitation energy of 2 MeV in ^{170}Yb which have not been discussed. Unfortunately, in this case the only one-neutron transfer process which can be studied with the use of stable targets, is the pick-up process, and a considerable ambiguity exists concerning the interpretation of the data. For the largest unassigned peaks in the (d,t) spectrum of Fig. 1, several possible combinations of rotational bands exist. For instance, the state at 1473 keV could be a $K\pi = 1+$, $3/2-[521] - 1/2-[521]$ configuration or a $K\pi = 0+$ state with a large $1/2-[521] - 1/2-[521]$ component similar to the levels discussed in section 4.4. The level at 1564 keV could be the spin 3 member of the possible $K\pi = 1+$ band mentioned above, or a $K = 2+$ state which contains the remainder of the $1/2-[521] + 3/2-[521]$ strength which was not found in the γ-vibration. Considering energies and cross sections at only one angle, one cannot exclude any of the above suggestions. However, (d,t) spectra have been recorded at four different angles for this particular reaction, and considerations of the cross section ratios at these angles suggest that the population of the 1564 keV state takes place with an l-value of three or more, whereas that of the 1473 keV level involves less than three units of angular momentum transfer. This would eliminate the assignment of spin 2 to the 1564 keV level and thus favour the choice of a $K\pi = 1+$ band at 1473 keV. The levels at 1473, 1508, 1564, and 1655 keV could be spin 1, 2, 3, and 4 rotational members. If this state were the $3/2-[521]-1/2-[521]$ configuration, the intensities for these four states would be expected to be 87, 27, 74, and 50 $\mu b/sr$, respectively, at $\theta = 90°$. The observed cross sections are 56, ~6, 45, and 16 $\mu b/sr$, respectively, which indicates that, if this is a $K\pi = 1+$ band, it consists of only slightly more than half of the $3/2-[521]-1/2-[521]$ state.

In view of the uncertainty of the nature of the strongly populated states discussed above, there is little use in making speculations concerning the weaker peaks. Many levels are expected in this region of excitation which can be populated by the transfer of $7/2+[633]$ or $5/2-[523]$ neutrons. As these orbitals have low values of $C_{jl}^2\varphi_l$, many weakly populated states are expected, which may be the explanation for many of the small peaks in Fig. 1.

4.6. Other Levels in ^{172}Yb

There has been a great deal of discussion concerning the $K\pi = 3+$ level at 1172 keV in ^{172}Yb. GALLAGHER and SOLOVIEV[20)] have considered the early experimental information and assigned the level to be the $K = 3+$, $1/2-[521]+5/2-[512]$ two-quasiparticle state. More recently, GÜNTHER et al.[26)] have measured the magnetic dipole moment of this state and concluded that only the $1/2-[521]-7/2-[514]$ two-neutron configuration was consistent with their results. The present measurements show clearly that the $1/2-[521]-5/2-[512]$ state is indeed present at this excitation energy. This is seen most easily from the data in Table 11 which compare the predicted cross sections for the two configurations with the experimental values. The state formed by coupling the $1/2-[521]$ and $5/2-[512]$ neutrons can be populated by both the (d,p) and (d,t) reactions because these orbitals are the ground states of the ^{171}Yb and ^{173}Yb target nuclei, respectively. However, the $1/2-[521]-7/2[514]$ state is not expected to be populated by the (d,t) reaction because neither component of this two-quasiparticle configuration is present in the ground state of ^{173}Yb. The $1/2-[521]+5/2-[512]$ state is the only one which should occur at such a low excitation energy and have such high cross sections in the (d,t) process.

A closer look at these data shows that the experimental (d,t) cross sections are only about 75 $^0/_0$ of the predicted values. Although the expected uncertainty on each of these values is about 15 $^0/_0$, it is noted that the experimental (d,p) cross section is also less than the predicted value by about the same factor. This could be an indication that only about three quarters of the state at 1172 keV is made up of the 1/2 − [521] + 5/2 − [512] combination.

It is interesting to speculate on the significance of the measured value of the magnetic moment. For instance, it has been suggested that both of the two-quasiparticle states mentioned above might be present at almost the same excitation energy near 1172 keV. This, of course, would make no difference to the (d,t) measurements and would make only a small difference for the (d,p) results as the transfer of a 7/2 − [514] neutron takes place with a cross section much smaller than that of a 5/2 − [512] neutron. Although the experimental (d,p) cross section to the band agrees more closely with that expected for the 1/2 − [521] + 5/2 − [512] state alone than with that expected for the sum of the two states, the agreement is not good enough to exclude the possibility that both bands are present at almost the same energy.

Another possibility is that the 1172 keV level might have some admixtures of two-proton states. The measured magnetic moment is 0.64 ± 0.04 n.m. The calculated magnetic moments for several two-quasiparticle states are as follows[26]

Two-neutron 1/2 − [521] + 5/2 − [512] $\mu = -0.126$ n.m.
1/2 − [521] − 7/2 − [514] $\mu = +0.59$ n.m.
Two-proton 7/2 + [404] − 1/2 + [411] $\mu = +2.19$ n.m.
5/2 + [402] − 1/2 + [411] $\mu = +2.87$ n.m.

The two-proton states listed consist of orbitals near the Fermi surface and are expected to have rather low excitation energies (1.4 and 1.7 MeV[20]). If the 1172 keV state contains only about three quarters of the 1/2 − [521] + 5/2 − [512] state as discussed above, and if the other 1/4 of the state is one of the two-proton configurations, a magnetic moment of ∼ + 1/2 n.m. could easily be explained.

This suggestion implies that the 1172 keV state has an admixture of more than one two-quasiparticle state, somewhat similar to the vibrational states discussed above, though with the amplitude of the 1/2 − [521] + 5/2 − [512] being ∼0.9. The inelastic deuteron scattering results may indicate a slightly collective nature for this state. It is seen that in the (d,d') population of vibrational levels only the transitions to the states of natural parity in the rotational bands are observed. In the $^{172}\mathrm{Yb}(d,d')$ spectrum, the 1263 keV level, which is the 4+ member of the rotational band based on the 1172 keV state, is seen. The exact mechanism of this process is not understood, but it is interesting to consider the possibility that small admixtures of other two-quasiparticle states could give rise to a collective hexadecapole moment which could cause the excitation of the 4+ state from the ground state by a direct E4-transition.

It was shown in section 4.2. that only about 25 % of the $K\pi = 2+$ two-quasiparticle state 5/2 – [512] – 1/2 – [521] was involved in the γ-vibration at 1466 keV. This means that a large fraction of this state should be observed elsewhere in the spectrum. The state under discussion is expected to have a large cross section, and one could perhaps make an attempt to associate it with some of the large peaks observed in the spectra. However, on the basis of the present stripping and pick-up reactions alone, it is not easy to identify this band with certainty, since it is difficult to decompose the observed spectra into clearly defined rotational bands. If this state were at an excitation of ~1.6 MeV, the 2+, 3+, and 4+ members of the rotational band would be expected to have cross sections of 133, 48, and 28 $\mu b/sr$ in the (d,t) reaction and 2, 48, and 17 $\mu b/sr$ in the (d,p) reaction, respectively, assuming an admixture of only 75 % for the 1/2 – [521] – 5/2 – [512] component. There is no unique group of levels which meets all the requirements exactly, but there are two sets of levels which fit roughly into bands and have cross sections comparable to those expected. Three states at approximately 1607, 1699, and 1803 keV have cross sections of 167, ~34, and ~30 $\mu b/sr$ in the (d,t) reaction and ~3, 33, and ~6 $\mu b/sr$ in the (d,p) process, respectively. Another set of levels found at 1660, 1750, and 1855 keV has (d,t) cross sections of 158, ~14, and ~7 $\mu b/sr$ and (d,p) cross sections of 20, 32, and ~3 $\mu b/sr$, respectively. As stated above, these states do not fit exactly the expectations, but it is noted that they are the only ones with reasonable intensities for the state in question. The levels at 1607 and 1660 keV are the only ones with (d,t) cross sections greater than ~20 $\mu b/sr$ until an excitation energy of ~2.5 MeV is reached. The peak in the (d,t) spectrum corresponding to a level at 1660 keV is broader than others nearby and thus probably includes more than one group. Therefore the first set of levels mentioned above is in better agreement with expectations than the second. However, there are also other reasons for choosing the 1607 keV band for the $K\pi = 2+$ band in question.

If there is some of the $K\pi = 2+$ two-quasiparticle state 5/2 – [512] – 1/2 – [521] mixed into the γ-vibration, some of the γ-vibration could be mixed into the two-neutron state. Hence, one might expect a small (d,d') population for the spin 2 member of the band at 1607 keV. It is seen that there is a small peak in the (d,d') spectrum at about this energy, which supports the assignment. It is also noted that Otteson[10) has studied the ^{172}Tm decay and found a $K\pi = 2+$ band where the $2+$, $3+$, and $4+$ members coincide, within errors, with those discussed above. On the basis of transitions seen in the decay, this band was assigned to be the $5/2-[512]-1/2-[521]$ configuration. The rotational energies in the band deviate considerably from the simple $I(I+1)$ dependence, but this might not be too surprising when one considers that there are several rotational bands with positive parity in this region of excitation energies.

There are four triton groups in the spectrum of Fig. 3, which correspond to levels at energies of 2008, 2046, 2109, and 2193 keV. These groups resemble the pattern expected for the $K\pi = 1+$, $5/2-[512]-3/2-[521]$ band for which the spin 1, 2, 3, and 4 members have predicted cross sections of 50, 50, 47, and 40 $\mu b/sr$, respectively. The observed intensities are 12, ~11, 20, and <14 $\mu b/sr$ and thus, if this set of levels is due to the configuration suggested, only a fraction of the single-particle strength is present.

As seen in the spectrum of Fig. 3, there are many strongly populated states above an excitation energy of 2 MeV. It is reasonable to expect that some of the lowest of these are states formed by stripping a $1/2-[510]$ or $3/2-[512]$ neutron, as these are the next lowest Nilsson states which have large values of $C^2_{jl}\varphi_l$. However, if one tries to compare the observations with predictions, it immediately becomes clear that the situation is complex. For instance, the stripping of a $1/2-[510]$ neutron should yield a $K\pi = 1+$ band with cross sections for the $1+$, $2+$, $3+$, and $4+$ members of approximately 45, 150, 70, and 30 $\mu b/sr$, respectively, and a $K\pi = 0+$ band whose $0+$, $1+$, $2+$, $3+$, $4+$, $5+$, and $6+$ spin members should have cross sections of 2.5, 120, 135, 34, 14, 1, and 0.2 $\mu b/sr$, respectively. The transfer of neutrons into states which lie higher in the Nilsson scheme takes place with similar, or larger, cross sections. However, the large peaks in Fig. 3 corresponding to ~2.5 MeV excitation have cross sections of 50 to 80 $\mu b/sr$ and are much too weak to represent a pure state of this nature. Hence, it appears that a situation similar to that observed in the odd Yb nuclei[2) prevails, whereby the states become strongly mixed and the intensity for a particular transfer process is distributed over several levels, resulting in a corresponding increase in the density of levels populated.

4.7. Other Levels in ^{174}Yb

The data pertaining to ^{174}Yb are more ambiguous than those for ^{172}Yb because only the (d,d') and (d,p) reactions were used. The ground state of ^{173}Yb is the $5/2-[512]$ orbital and thus all two-quasiparticle states in ^{174}Yb which are populated by the (d,p) reaction involve this neutron. The stripping cross sections to rotational bands based on states at 0, 1320, and 1630 keV have already been discussed. Another level at ~1518 keV has previously been observed[11, 12, 13] in the decays of ^{174}Lu and ^{174}Tm, and there are differences of opinion as to whether it is the $K\pi = 6+$, $5/2-[512]+7/2-[514]$ or the $K\pi = 7-$, $5/2-[512]+9/2+[624]$ state. One positive piece of information which favours the first choice is that the 994 keV gamma transition leading to the $6+$ member of the ground state rotational band has an internal conversion coefficient which indicates that its multipolarity is E2[13]. However, it is possible that both states may exist at ~1520 keV excitation and are populated differently in the two decay processes.

If the $6+$ assignment is correct, one would expect to observe (d,p) cross sections to the spin 6 and 7 members of the rotational band of approximately 20 and 16 $\mu b/sr$, respectively. There is a proton group populating a state at ~1509 keV with an intensity of 17 $\mu n/sr$, which could be the $6+$ state, although the energy deviation from the previous value is slightly larger than the expected experimental error. The proton group populating the level at ~1667 keV has been partly ascribed to the spin 6 member of the band based at 1320 keV. However, there is ≥ 10 $\mu b/sr$ of this peak left unassigned, which could be due to the spin 7 member of the $K\pi = 6+$ band. This choice would correspond to an inertial parameter $\hbar^2/2\mathfrak{J}$ of ~11.5 keV, which would be quite reasonable for the $5/2-[512]+7/2-[514]$ state.

If the $K\pi = 7-$ state were found at ~1520 keV, one would expect (d,p) cross sections to the spin 7, 8, and 9 members of ~5, 10, and 16 $\mu b/sr$, respectively. If the peak at ~1723 keV were due to the spin 8 member and part of that at 1947 keV due to the spin 9 member, the value of $\hbar^2/2\mathfrak{J}$ would be ~12.5 keV, which is rather high for this two-quasiparticle state. One cannot, however, exclude the possibility that the spin 8 member is unresolved from the strongly populated state at 1702 keV and the spin 9 member is included in the peak at ~1926 keV. Thus it is seen that the (d,p) data give some support to the assignment of the $K\pi = 6+$ state at ~1520 keV, but do not exclude the possibility that the $K\pi = 7-$ state may be present at about the same excitation energy.

It was seen that about half the intensity of the $K\pi = 2+$, $5/2-[512]-1/2-[510]$ two-quasiparticle state was found in the γ-vibration at 1630 keV. This combination involves the stripping of a $1/2-[510]$ neutron which occurs with large cross sections. It should therefore be easy to find the remainder of the strength to the $K\pi = 2+$ band and also to the $K\pi = 3+$ band which is formed by coupling the same two neutrons with their spins parallel to each other. In the spectrum of Fig. 4, a number of strong proton groups are seen with excitation energies between 2.1 and 2.5 MeV. The expected cross sections for a pure $K = 2$ state formed by stripping a 1/2 – [510] neutron in this region of excitation energies are 111, 129, and 69 $\mu b/sr$ for the spin 2, 3, and 4 members, respectively. The $K = 3$ band would be expected to have cross sections of 154, 133, and 41 $\mu b/sr$ to the 3, 4, and 5 spin members, respectively. As about half of the strength of the $K = 2$ state is found in the γ-vibration, the $K = 3$ band is expected to have the largest peaks. It is seen that none of the peaks is quite as large as the predicted intensities for the $K = 3$ band, but the states at 2284, 2370, and 2482 keV have cross sections of 95, 67, and 26 $\mu b/sr$, respectively. These levels probably represent the 3, 4, and 5 spin members of a band which contains about 60 % of the $K\pi = 3+$, $5/2-[512]+1/2-[510]$ configuration.

There are remaining peaks corresponding to states at 2189, 2237, and ~2333 keV which have an intensity pattern resembling that of the spin 2, 3, and 4 members based on the γ-vibration. The absolute cross sections to these levels are consistent with those one would expect for a band containing approximately one-half of the $5/2-[512]-1/2-[510]$ state. Further evidence for this suggested assignment is presented by the (d, d') spectrum which, in analogy to ^{172}Yb, shows small peaks at the excitation energies of the proposed spin 2 and 4 members.

The assignments discussed in this section must be regarded as being rather tentative. Some of the ambiguities coud be removed by a careful study of the angular distributions of the strong peaks preferably with better resolution.

Summary

The low-lying levels in even-even Yb nuclei have been studied by the single-neutron transfer processes (d, p) and (d, t), and by the inelastic scattering process (d, d'). The characteristics of these reactions are fairly well known from studies on other nuclei and therefore the main effort in the present work was directed toward the extraction of a maximum amount of information about the levels in the even Yb nuclei from the observed cross sections. Particular emphasis has been placed on the decomposition of the nuclear states in terms of two-quasiparticle excitations. For this purpose it was important that the collective states built on the ground state could be identified from the (d, d') experiments.

In some cases, the absolute values of the cross sections for stripping

reactions leading to the ground state in the even nuclei did deviate from those calculated on the basis of the cross sections for the inverse transfer process leading to the odd nucleus. The origin of this descrepancy is not clear, but it might partly be connected with imperfections in the DWBA procedure used in the comparison of the cross sections. In other cases, however, the populations of ground state $K = 0$ bands yielded values of $U^2(\nu)$ and $V^2(\nu)$ for orbitals involved in the paired ground state, which were in good agreement with the corresponding values obtained from reactions on even target nuclei.

The (d,d') reaction systematically populated the $2+$ and $4+$ members of the $K = 2$ quadrupole vibrations, although the reduced transition probabilities were quite low. In a few cases, a weak population by (d,d') was observed for bands with $K\pi = 0+$. One of the most interesting aspects of the present work is that the (d,p) and (d,t) cross sections to the vibrational bands confirm the validity of considering such states to consist of superpositions of two-quasiparticle states. The present results lend convincing support to the calculated decomposition[17, 18] of the vibrations, although the predicted amplitudes often show only qualitative agreement with the measurements.

In all the nuclei, states were observed in the (d,d') reaction which were assigned as octupole vibrations. The collective octupole strength, however, appears to be weak in the Yb-region. In ^{174}Yb, it was possible to identify a two-quasiparticle component in one of the octupole bands, which indicated that the octupole state with lowest energy has $K\pi = 2-$.

In ^{172}Yb, the nature of several states has been definitely established. These include states with $K\pi = 0+$ at 1045 keV, $K\pi = 3+$ at 1172 keV, and $K\pi = 2+$ at 1607 keV. In addition, several other states whose assignments are not so certain have been observed, including a possible $K\pi = 1+$ level at 2008 keV. In ^{170}Yb and ^{174}Yb, several levels have been tentatively assigned as two-quasiparticle states, sometimes with the assistance of previous investigations. It was found that the possibility of using both neutron transfer reactions to populate levels in ^{172}Yb was of considerable importance in eliminating ambiguities in the assignments and, therefore, it was more difficult to study the levels in ^{170}Yb and ^{174}Yb. For these nuclei it would be very helpful to carry out a detailed study of the angular distributions, preferably using better resolution than was employed in the present work.

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